An Object-Logic Explanation for the Inconsistency in Gödel's Ontological Theory (Ext. Abstract — IJCAI-16 Paper)

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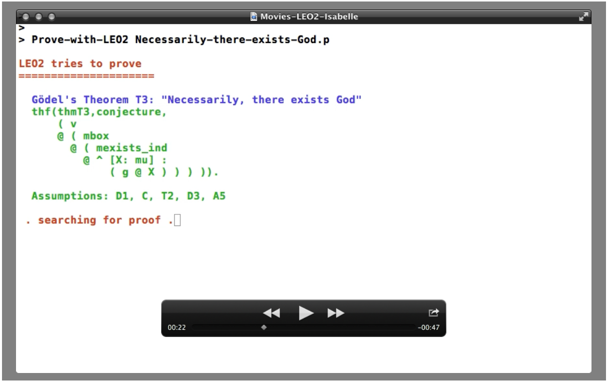
KI, Klagenfurt, September 28, 2016



Url to movie: `http://www.christoph-benzmueller.de/papers/Movies-LEO2-Isabelle.mov`

[1] Supported by DFG Heisenberg Fellowship BE 2501/9-1/2

Vision of Leibniz (1646–1716): *Calculemus!* (Formal Analysis of Arguments)

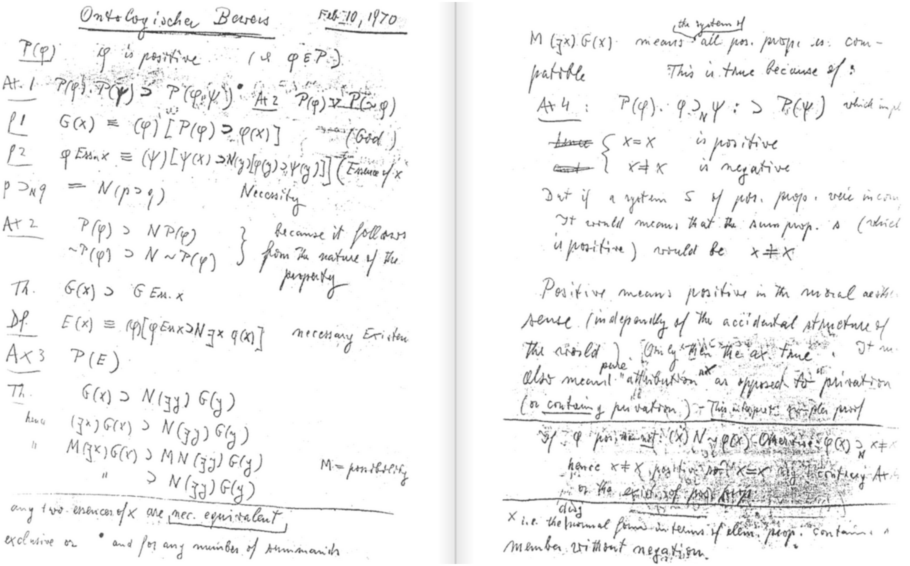
If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, to sit down to their slates, and to say to each other . . . : Let us calculate.

(Translation by Russell)

Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo . . . dicere: calculemus. (Leibniz, 1684)

Required:
characteristica universalis and **calculus ratiocinator**

Kurt Gödel's Manuscript: 1930's, 1941, 1946-1955, 1970



Ontologischer Beweis Feb 10, 1970

$P(\varphi)$ φ is positive (or $\varphi \in P$)

At. 1 $P(\varphi).P(\psi) \supset P(\varphi.\psi)$ At 2 $P(\varphi) \vee P(\sim\varphi)$

P1 $G(x) \equiv (\varphi)[P(\varphi) \supset \varphi(x)]$ (God)

P2 $\varphi \text{ Ess. } x \equiv (\psi)[\psi(x) \supset N(y)[\varphi(y) \supset \psi(y)]]$ (Essence of x)

$p \supset_N q = N(p \supset q)$ Necessity

At 2 $P(\varphi) \supset NP(\varphi)$, $\sim P(\varphi) \supset N\sim P(\varphi)$ } because it follows from the nature of the property

Th. $G(x) \supset G \text{ Ess. } x$

Df. $E(x) \equiv (\varphi)[\varphi \text{ Ess } x \supset N \exists x\, \varphi(x)]$ necessary Existence

Ax 3 $P(E)$

Th. $G(x) \supset N(\exists y)G(y)$

hence $(\exists x)G(x) \supset N(\exists y)G(y)$

" $M(\exists x)G(x) \supset MN(\exists y)G(y)$

" $\supset N(\exists y)G(y)$ M = possibility

any two essences of x are nec. equivalent

exclusive or • and for any number of summands

$M(\exists x)G(x)$ means the system of all pos. props. is compatible. This is true because of:

At 4: $P(\varphi).\varphi \supset_N \psi : \supset P(\psi)$ which impl

{ $x=x$ is positive, $x \neq x$ is negative

But if a system S of pos. props. were incom It would mean that the sum prop. s (which is positive) would be $x \neq x$

Positive means positive in the moral aesth. sense (independently of the accidental structure of the world). Only then the ax. true. It m also meant pure "attribution" as opposed to "privation" (or containing privation). This interpret. simpler proof

If φ pos. not: $(x) N\sim\varphi(x)$. Otherwise: $\varphi(x) \supset_N x \neq x$ hence $x \neq x$ positive not $x=x$ neg. contrary Ax or the existence of pos. prop.

X i.e. the disj normal form in terms of elem. prop. contains a member without negation.

Dana Scott's slightly modified version of Gödel's Argument (1972)

Axiom A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

Axiom A2 A property necessarily implied by a positive property is positive:
$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

Thm. T1 Positive properties are possibly exemplified: $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$

Def. D1 A *God-like* being possesses all positive properties: $G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$

Axiom A3 The property of being God-like is positive: $P(G)$

Cor. C Possibly, God exists: $\Diamond\exists x G(x)$

Axiom A4 Positive properties are necessarily positive: $\forall\phi[P(\phi) \rightarrow \Box P(\phi)]$

Def. D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties: $\phi\ ess.\ x \leftrightarrow \phi(x) \wedge \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

Thm. T2 Being God-like is an essence of any God-like being: $\forall x[G(x) \rightarrow G\ ess.\ x]$

Def. D3 *Necessary existence* of an individual is the necessary exemplification of all its essences: $NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$

Axiom A5 Necessary existence is a positive property: $P(NE)$

Thm. T3 Necessarily, God exists: $\Box\exists x G(x)$

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Difference to Gödel (who omits this conjunct)

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Modal operators are used

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higher-order quantifiers

Challenge

Formal analysis of nontrivial arguments in metaphysics requires the implementation of (variants of)

Higher-Order Modal Logics

or generally

Expressive Non-Classical Logics

on the computer.

Inexpressive logics are useless here!

HOL as a Universal (Meta-)Logic via Semantic Embeddings

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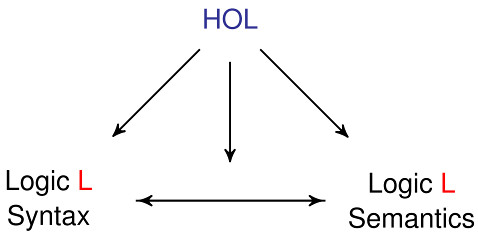


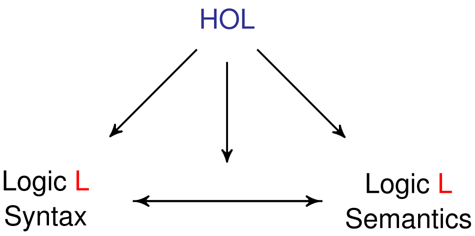
Examples for L we have already studied:
Modal Logics, Description Logics, Conditional Logics, Intuitionistic Logics, Access Control Logics, Nominal Logics, Multivalued Logics (SIXTEEN), Logics based on Neighborhood Semantics, (Mathematical) Fuzzy Logics, Paraconsistent Logics, Free Logic . . .

Embedding works also for quantifiers (first-order & higher-order)

HOL provers become universal logic reasoning engines!

interactive:	Isabelle/HOL, PVS, HOL4, Hol Light, Coq/HOL, . . .
automated:	TPS, LEO-II, Satallax, Nitpick, Isabelle/HOL, . . .

HOL as a Universal (Meta-)Logic via Semantic Embeddings



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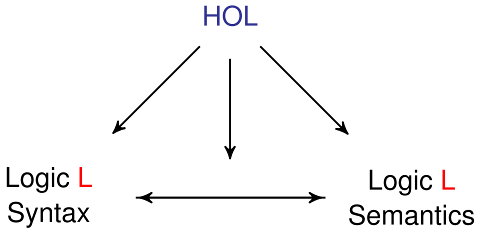

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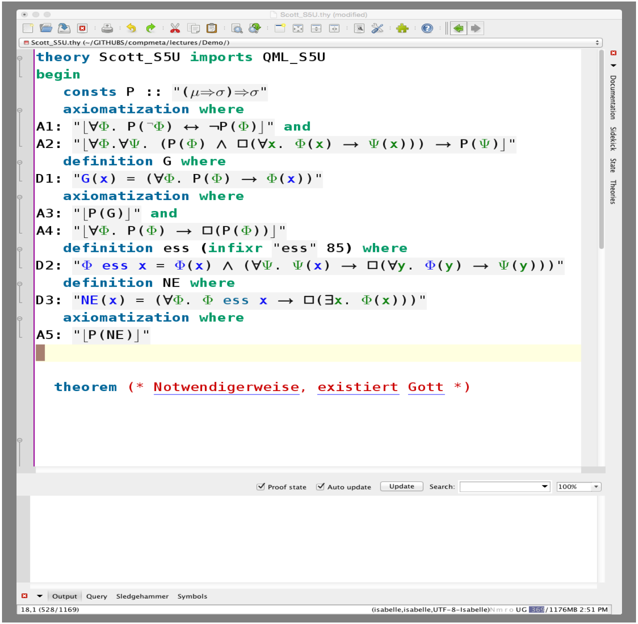
Formalising and Analysing Gödel's God in Isabelle/HOL



Url to movie: `http://www.christoph-benzmueller.de/papers/DemoMovieLehrpreis2.mov`

Inconsistency (Gödel): Proof by LEO-II in Modal Logic KB

DemoMaterial — bash — 166×52

```
@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: $false))@SV3)=$true))),inference(prim_subst,[status(thm)],[66:[bind(SV11,$thf(^[SV23:mu,SV24:$i]: $false))]])).
 thf(84,plain,(![SV22:(mu>($i>$o)),SV3:$i,SV8:(mu>($i>$o))]: ((((SV8@(((sK2_SY33@SV3)@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV8))@(((sK1_SY31@(^[SX0:mu,SX1:$i]: (
~ ((SV22@SX0)@SX1))))@SV8)@SV3))=$true) | (((p@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV3)=$true))),inference(prim_subst,[status(thm)],[66
:[bind(SV11,$thf(^[SV20:mu,SV21:$i]: (~ ((SV22@SV20)@SV21))))]])).
 thf(85,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false) | ((((p@SV9)@SV4) = ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)
@SY28))))@SV4))=$false))),inference(fac_restr,[status(thm)],[56])).
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SY30))))@SV4))=$false))),inference(fac_restr,[status(thm)],[57])).
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SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)))))=$false) | (((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_equal_neg,[status(thm)],[85])).
 thf(89,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((~ (((p@SV9)@SV4) | ((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4))) | (~ ((~ ((p@SV9)@SV4)) | (~ ((p@(^[SY29:mu,
SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)))))=$false) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true))),inference(extcnf_equal_neg,[status(thm)],[86])).
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~ ((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_or_neg,[status(thm)],[87])).
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SY29)@SY30))))@SV4)=$true))),inference(extcnf_or_neg,[status(thm)],[89])).
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V9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_not_neg,[status(thm)],[92])).
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@SY30))))@SV4)=$true))),inference(extcnf_not_neg,[status(thm)],[93])).
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 (~ ((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_or_pos,[status(thm)],[96])).
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SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_not_pos,[status(thm)],[103])).
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 (~ ((SV22@SX0)@SX1))))@SV8)@SV3))=$true) | (((p@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV3)=$true))),inference(extcnf_not_neg,[status(thm
)],[78])).
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SX1:$i]: (~ ((SV15@SX0)@SX1))))@SV3))=$false) | (((p@(^[SX0:mu,SX1:$i]: (~ ((SV15@SX0)@SX1))))@SV3)=$false) | (((p@SV11)@SV3)=$true))),inference(extcnf_not_pos,[statu
s(thm)],[81])).
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5])).
 thf(110,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@SV9)@SV4)=$true) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true))),inference(sim,[status(thm)],[101]
)).
 thf(111,plain,(![SV3:$i,SV8:(mu>($i>$o))]: ((((p@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: $true))@SV3)=$true))),inference(sim,[status(thm)],[76])).
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 thf(114,plain,($false),inference(solved_all_splits,[solved_all_splits(join,[])],[113])).
% SZS output end CNFRefutation

%***** End of derivation protocol ****
%***** no. of clauses in derivation: 97 ****
%***** clause counter: 113 ****

% SZS status Unsatisfiable for ConsistencyWithoutFirstConjunctinD2.p : (rf:0,axioms:6,ps:3,u:6,ude:false,rLeibEQ:true,rAndEQ:true,use_choice:true,use_extuni:true,use_
extcnf_combined:true,expand_extuni:false,foatp:e,atp_timeout:25,atp_calls_frequency:10,ordering:none,proof_output:1,clause_count:113,loop_count:0,foatp_calls:2,transl
ation:fof_full)
ontoleo:DemoMaterial cbenzmueller$
```

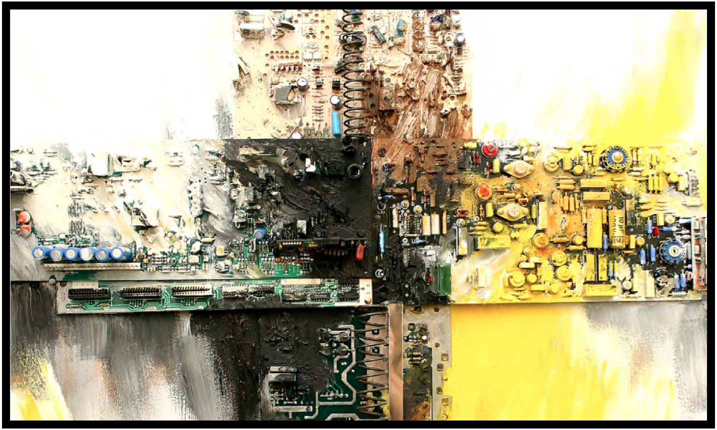

Reconstruction of the Inconsistency of Gödel's Axioms

See our IJCAI-16 paper

Inconsistency (Gödel): Reconstructed Argument (in Modal Logics KB and K)

[Benzmüller&Woltzenlogel-Paleo, IJCAI, 2016]

Def. D2* $\phi\ ess.\ x \leftrightarrow \cancel{\phi(x) \wedge}\ \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

Lemma 1 The empty property is an essence of every entity. $\forall x\,(\emptyset\ ess.\ x)$

Theorem 1 Positive Properties are possibly exemplified. $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$

Axiom A5 $P(NE)$

- by T1, A5: $\Diamond\exists x[NE(x)]$

Def. D3 $NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$

- $\Diamond\exists x[\forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y[\phi(y)]]]$
- $\Diamond\exists x[\emptyset\ ess.\ x \rightarrow \Box\exists y[\emptyset(y)]]$
- by L1 $\Diamond\exists x[\top \rightarrow \Box\exists y[\emptyset(y)]]$
- by def. of $\emptyset$ $\Diamond\exists x[\top \rightarrow \Box\bot]$
- $\Diamond\exists x[\Box\bot]$
- $\Diamond\Box\bot$

Inconsistency $\bot$

The last step is not hard to justify semantically: we did this in the IJCAI-16 paper!
A syntactical proof is not entirely trivial: novel contribution to KI-2016

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[Benzmüller&Woltzenlogel-Paleo, IJCAI, 2016]

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$\Diamond\Box\bot$ Implies Inconsistency: Syntactical Proof in Modal Logic K

- Syntactical proof in proof assistent Coq
- Assume: $\Diamond\Box\bot$ (holds globally)
- Show: there exist a formula φ (in current world) s.t. φ and $\neg\varphi$
- Choose $\varphi := \Diamond\Diamond\bot$
- $\Diamond\Diamond\bot$ follows from $\Diamond\Box\bot$
- $\neg\Diamond\Diamond\bot$ holds: $\neg\Diamond\Diamond\bot \equiv \Box\Box\top$, which easily follows by necessitation
- Hence, $\Diamond\Diamond\bot$ and $\neg\Diamond\Diamond\bot$
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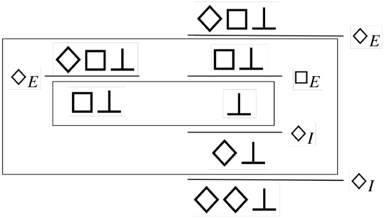

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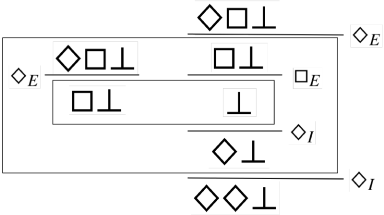

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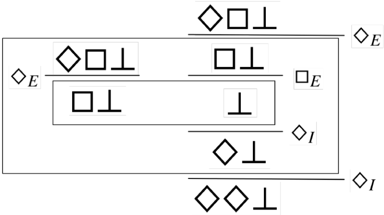

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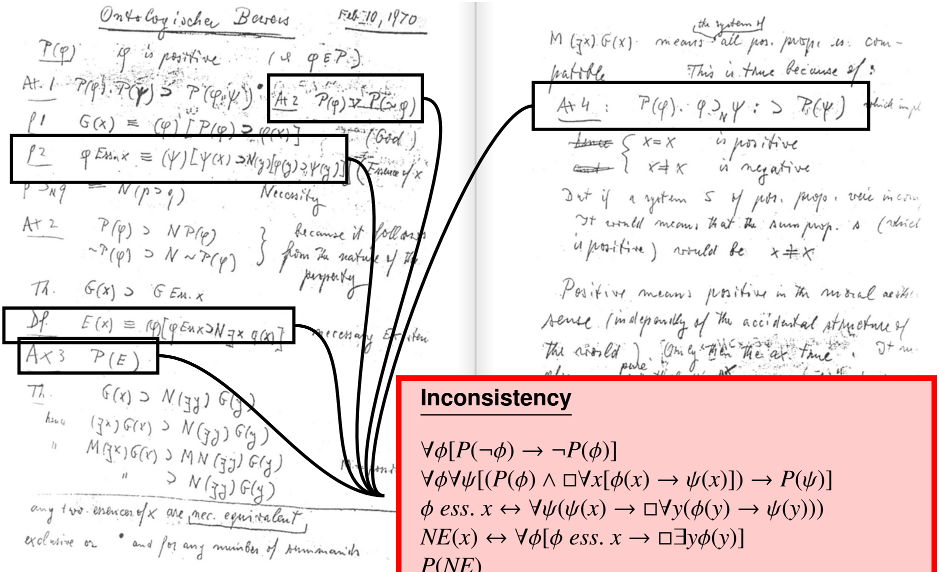
Gödel's Manuscript: Identifying the Inconsistent Axioms



Ontologischer Beweis Feb 10, 1970

P(φ) φ is positive (i.e. φ ∈ P)

At. 1 P(φ).P(ψ) ⊃ P(φ.ψ) At 2 P(φ) ∨ P(~φ)

P1 G(x) ≡ (φ)[P(φ) ⊃ φ(x)] (God)

P2 φ Ess. x ≡ (ψ)[ψ(x) ⊃ N(y)[φ(y) ⊃ ψ(y)]] (Essence of x)

p ⊃_N q = N(p ⊃ q) Necessity

At 2 P(φ) ⊃ N P(φ)
~P(φ) ⊃ N ~P(φ) } because it follows from the nature of the property

Th. G(x) ⊃ G Ess. x

Df. E(x) ≡ (φ)[φ Ess x ⊃ N ∃x φ(x)] necessary Existence

Ax 3 P(E)

Th. G(x) ⊃ N(∃y) G(y)
hence (∃x) G(x) ⊃ N(∃y) G(y)
" M(∃x) G(x) ⊃ M N(∃y) G(y)
" ⊃ N(∃y) G(y)

any two essences of x are nec. equivalent

exclusive or * and for any number of summands

M(∃x) G(x) means the system of all pos. props. is compatible. This is true because of:

At 4 : P(φ). φ ⊃_N ψ : ⊃ P(ψ) which impl

{ x=x is positive
x≠x is negative

But if a system S of pos. props. were incom

It would mean, that the sum prop. s (which is positive) would be x≠x

Positive means positive in the moral aesth. sense (independently of the accidental structure of the world). Only then the ax. true. It m

Inconsistency

$$\forall \phi [P(\neg \phi) \rightarrow \neg P(\phi)]$$
$$\forall \phi \forall \psi [(P(\phi) \wedge \Box \forall x [\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$
$$\phi \; ess. \; x \leftrightarrow \forall \psi (\psi(x) \rightarrow \Box \forall y (\phi(y) \rightarrow \psi(y)))$$
$$NE(x) \leftrightarrow \forall \phi [\phi \; ess. \; x \rightarrow \Box \exists y \phi(y)]$$
$$P(NE)$$

Conclusion

Overall Achievements

- significant contribution towards a **Computational Metaphysics**
- **novel results** contributed by **HOL-ATPs**
- infrastructure can be adapted for **other logics and logic combinations**
- **basic technology works well**; further improvements are on the way

Relevance (wrt foundations and applications)

- Philosophy, AI, Computer Science, Computational Linguistics, Maths

Related work: only for Anselm's simpler argument

- first-order ATP PROVER9 [OppenheimerZalta, 2011]
- interactive proof assistant PVS [Rushby, 2013]

Ongoing work

- (Awarded) Lecture course **Computational Metaphysics** at FU Berlin
- Landscape of verified/falsified ontological arguments
- You may contribute: `https://github.com/FormalTheology/GoedelGod.git`